Bright Solitary Waves of Atomic Bose-Einstein Condensates

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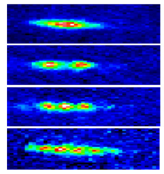
1. Introduction

Bright solitons are one-dimensional waves that propagate without dispersion. The analogous 3D structure, termed *bright solitary wave* (BSW), has been demonstrated in atomic Bose-Einstein condensates (BECs) with attractive interactions [1,2]. These self-trapped matter waves hold exciting possibilities for atom-optical applications, e.g., interferometry. However, the presence of 3D introduces complex additional behaviour, e.g., the collapse instability [3,4] and inelastic collisions [5].

A recent experiment at JILA [2] created multiple BSWs and observed surprising behaviour. Despite being in the collapse regime and an almost 3D system, the BSW dynamics, including many collisions, were remarkably stable. Here we study the intriguing properties of bright solitary matter waves.



2. Theoretical Framework

At ultra-cold temperatures, a BEC can be described by a mean-field wavefunction $\psi(\mathbf{r},t)$ which satisfies the Gross-Pitaevskii equation (GPE),

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{r})+\frac{4\pi\hbar^2 a_s}{m}|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t).$$

The nonlinearity, parameterised by the *s*-wave scattering length a_s, is assumed to be attractive ($a_s<0$).

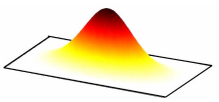
1D: For $V(\mathbf{r})=0$, exact bright soliton solutions are supported, $\psi(z)\sim\mathrm{sech}(z/\xi)$. Bright solitons can have any amplitude and undergo elastic collisions.

3D: We assume a confining potential $V(\mathbf{r})=\frac{1}{2}m\omega_r^2(r^2+\lambda^2z^2)$. Defining the interactions via $k=N|a_s|/a_r$ (N is the atom number and $a_r=(\hbar/m\omega_r)^{1/2}$) a 3D attractive BEC collapses when $k>k_c$ (or $N>N_c$).

3. Bright Solitary Waves and Collapse

(a) Variational approach

We assume an ansatz for the BSWs [3,4]:



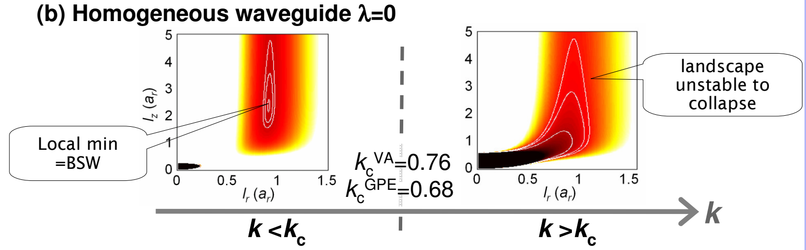
$$\psi(r,z)=\left(\frac{N}{2\pi l_z l_r^2}\right)^{1/2}\mathrm{sech}\left(\frac{z}{l_z}\right)\exp\left(-\frac{r^2}{2l_r^2}\right)$$

This defines an energy landscape in terms of widths l_r and l_z:

$$\frac{E_A}{N}=\frac{\hbar^2}{2m}\left(\frac{1}{l_r^2}+\frac{1}{3l_z^2}\right)+\frac{1}{2}m\omega_r^2\left(l_r^2+\frac{\pi^2\lambda^2 l_z^2}{12}\right)-\frac{a_r\hbar^2 k}{3ml_z l_r^2}$$

This variational approach gives excellent insight, e.g., solutions, regimes of collapse, excitation spectrum and energetic stability.

(b) Homogeneous waveguide $\lambda=0$



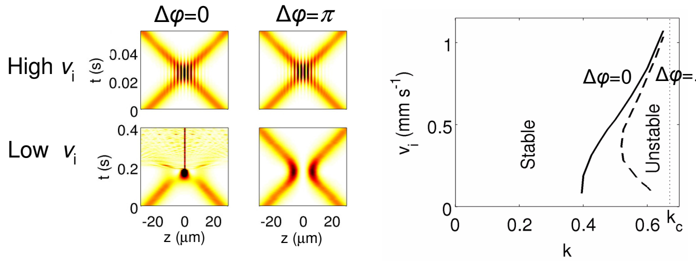
(c) Axial trapping $\lambda=0.4$

JILA experiment: $k_c=0.64\pm0.07$

Theory: $k_c^{GPE}=0.642$ $k_c^{VA}=0.72$

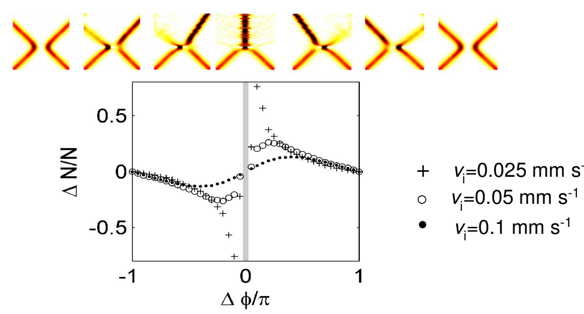
4. BSW Collisions in a Homogeneous System

(a) Phase differences $\Delta\varphi=0,\ \pi$



- Collisions prone to collapse, especially for $\Delta\varphi=0$
- $\Delta\varphi=\pi$ prevents overlap and suppresses collapse
- At high speed, collisions stable and independent of $\Delta\varphi$
- Stable when $t_{collision}<t_{collapse}$; unstable when $t_{collision}>t_{collapse}$

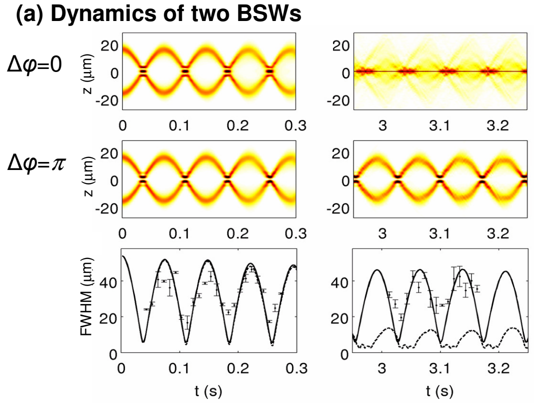
(b) Phase difference $-\pi\leq\Delta\varphi\leq\pi$



- Significant population transfer ΔN during collision
- Sensitive to $\Delta\varphi$: sinusoidal at high v_i, divergent at low v_i
- Route to solitonic interferometry

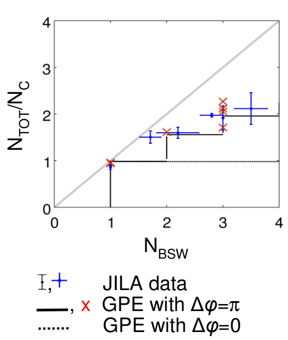
5. BSW Interactions and the JILA Experiment

(a) Dynamics of two BSWs



- $\Delta\varphi=\pi$ crucial to explain experimental observations
- Excellent agreement with experiment

(b) Multiple BSWs



6. Current Directions

- ***Dipolar BSWs***: Extending the variational approach to include dipolar atomic interactions reveals that BSWs are supported by purely dipolar interactions [6] and that they are more stable than their *s*-wave counterparts. This is confirmed by numerical solution of the dipolar GPE. Moreover, the anisotropy and long-range of the dipolar interactions is expected to lead to novel properties, particularly in their collisions.
- ***Quantum reflection with BSWs:*** Repulsive BECs have been quantum reflected from an attractive surface potential but, at low velocity, this was obscured by BEC disruption. BSWs are expected to give greatly enhanced quantum reflection, and may ultimately be exploited as ultra-precise surface probes [7].
- ***Rotating BSWs:*** We have modelled the rotation of BSWs by extending the variational approach to include frame rotation and an irrotational flow pattern $\mathbf{v}=\alpha\nabla(xy)$, similar to rotating repulsive BECs. Providing the rotation frequency $<\omega_r$ the critical point for collapse can be significantly extended [8].

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