Bright matter-wave solitons: Formation, dynamics and quantum reflection

S.Händel, A.L.Marchant, T.P.Wiles, S.A.Hopkins, C.S.Adams and S.L.Cornish
Department of Physics, Durham University, Durham DH1 3LE, UK

Bose-Einstein condensates in 1D can support both dark (local minima in the wave function) and bright (local maxima) solitons depending on whether the interactions are repulsive or attractive, respectively. Previously, we have demonstrated that a robust configuration of multiple solitons is created during the collapse of ^{85}Rb condensates[1]. Confined in a cylindrically symmetric trap, the solitons were observed to oscillate along the weaker axial direction repeatedly colliding in the trap centre. Detailed analysis of binary soliton collisions using the Gross-Pitaevaskii equation (GPE) shows that the stability of these collisions depend critically on the relative phase and velocity of the solitons[2]. Moreover, the GPE results suggest that the solitons must form with a relative phase of π to ensure their observed stability[2]. However, new theoretical work suggests that the inclusion of quantum fluctuations causes the soliton dynamics to be predominantly repulsive in 1D independent of their initial relative phase[3]. We report the development of a new apparatus designed to resolve this question and to explore the application of solitons in atom interferometry and precision measurement through the study of atom-surface interactions and quantum reflection.

Interactions in Bose-Einstein condensates

Atomic interactions in BEC are accurately described in a mean field model by a self interaction energy that depends only on the number density and the s-wave scattering length a. The macroscopic wavefunction satisfies a non-linear Schrödinger equation, known as the Gross-Pitaevskii equation (GPE):

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{r})+g|\Psi(\mathbf{r},t)|^2\right)\Psi(\mathbf{r},t) \qquad g=4\pi\hbar^2 a/m$$

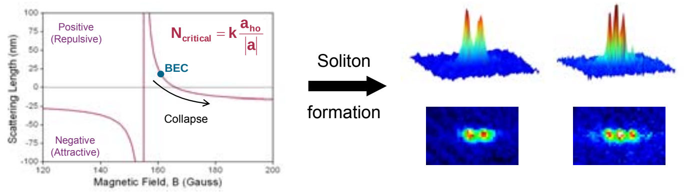
Bright soliton solution for attractive interactions (a < 0) in 1D:

$$\Psi(z,t)=\frac{1}{\sqrt{2l_z}}\operatorname{sech}\left(\frac{z-vt}{l_z}\right)\exp(-i\mu t/\hbar)$$

Healing length:

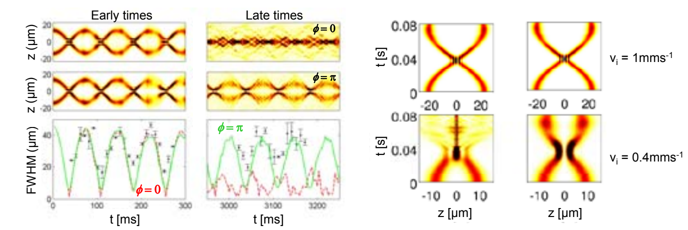
$$l_z=2\hbar(m|g_{1D}|N)$$

Feshbach resonances permit the precise control of the s-wave scattering length and can be used to trigger the collapse of the condensate[4] and the formation of bright matter-wave solitons[1,5].

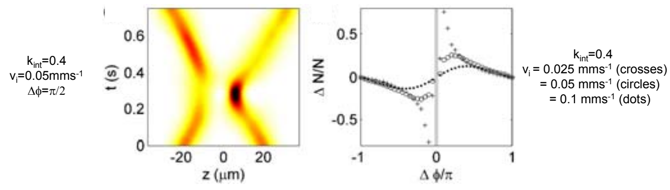


Binary soliton collisions: population transfer[2]

Previously, we created two solitons that oscillated along the weaker axial direction of a cylindrical trap repeatedly colliding in the trap centre[1]. Within the GPE model, the observed stability can only be explained by a π relative phase between the solitons[2].

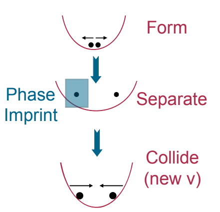


Intermediate phases show asymmetric density distributions and a transfer of population between solitons that depends sensitively on the relative phase and velocity[2].



→ Josephson-like tunneling → Soliton Interferometry?

Probing the relative phase: testing GPE theory



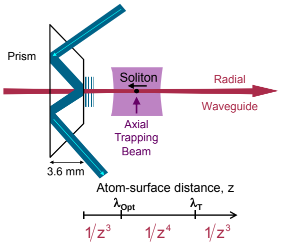
New theoretical work suggests that the inclusion of quantum fluctuations leads to predominantly repulsive soliton collisions independent of the initial relative phase[3].

Using an adjustable optical trapping potential to control the soliton velocity and phase imprinting[6] to alter the relative phase, we plan to engineer controlled binary collisions to

- **Test the GPE predictions for population transfer**
- **Settle debate over relative phase**

Probing atom surface interaction using solitons

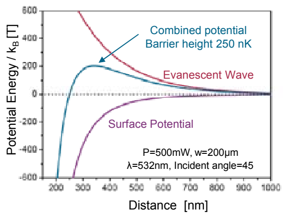
Measure classical reflection as a function of velocity and barrier height and position.



$z<\lambda_{opt}/2\pi$ van der Waals $U(z)=-\frac{C_3}{z^3}$

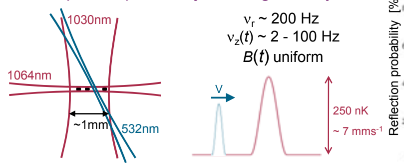
$\lambda_{opt}/2\pi<z<\lambda_T$ Casimir-Polder $U(z)=-\frac{C_4}{z^4}$

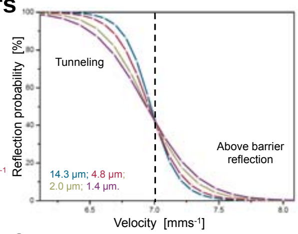
$z>\lambda_T\approx 7.6\,\mu m$ Thermal regime $U(z)=-\frac{k_B T\alpha_0}{4z^3}\frac{\varepsilon_0-1}{\varepsilon_0+1}$



Reflection from Gaussian Barriers

Crossed dipole trap with adjustable geometry:





Quantum reflection from solid surface

Quantum reflection requires the local wave vector to change by more than k over a distance of 1/k:

$$\frac{1}{k^2}\frac{dk}{dr}>1 \qquad k=\sqrt{k_\infty^2-2mU(z)/\hbar}$$

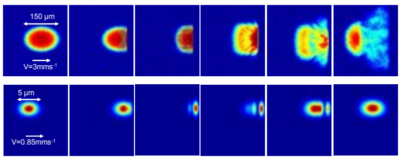
For a surface, in the low energy limit ($k\to 0$)[7]: $R\approx 1-2\beta_4 k$ $\quad C_4=\frac{\beta_4^2\hbar}{2m}$

Traditional experiments achieve appreciable reflection by:

Weak potential: H,He on He surface[8,9]

Low velocity: Grazing incidence[10]

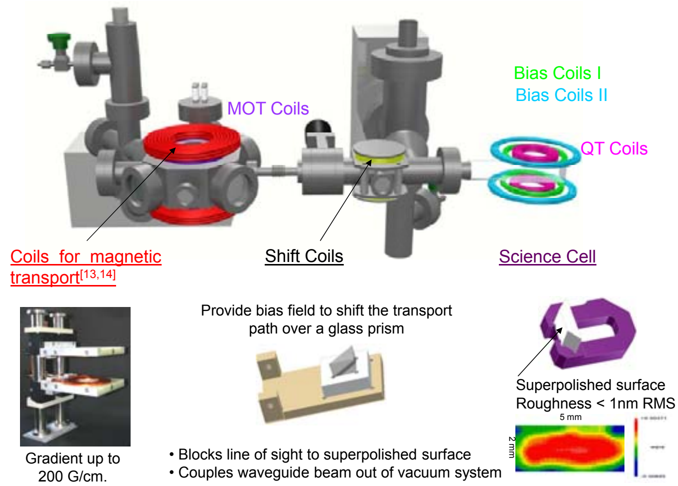
Ultracold atoms offer new possibilities



Quantum reflection of a ^{23}Na BEC on a Si surface. After reflection the BEC is disrupted[11].

Simulations of solitons predict a clean reflection without changing of shape[12].

Experimental apparatus



Gradient up to 200 G/cm.

- Blocks line of sight to superpolished surface
- Couples waveguide beam out of vacuum system

Collaborators: Andy Martin, Melbourne University, Australia. Nick Parker, McMaster University, Canada. Mark Fromhold, Nottingham University, UK.

References
[1] S.L. Cornish *et al.*, Phys. Rev. Lett. **96**, 170401 (2006).
[2] N.G. Parker *et al.*, J. Phys. B. **41**, 045303 (2008).
[3] B. J. Dąbrowska-Wüster, S. Wüster, and M. J. Davis, ArXiv:0812.0493.
[4] J.L. Roberts *et al.*, Phys. Rev. Lett. **86**, 4211 (2001).
[5] K. E. Strecker *et al.*, Nature **417**, 150 (2002); L. Khaykovich et al., Science 296, 1290 (2002).
[6] J. Denschlag *et al.*, Science **287**, 97 (2000).
[7] H. Friedrich et al., PRA. **65**, 032902 (2002).
[8] J. J. Berkhout *et al.*, PRL. **63**, 1689 (1989).
[9] V. U. Nayak *et al.*, PRL. **50**, 990 (1983).
[10] F. Shimizu *et al.*, PRL. **86**, 987 (2001).
[11] T. A. Pasquini *et al.*, Phys. Rev. A. **64**, 053605 (2006).
[12] S. L. Cornish *et al.*, http://dx.doi.org/10.1016/j.physd.2008.07.011 (22.09.2008).
[13] H. J. Lewandowski *et al.*, J. Low Temp. Phys. **132**, 309 (2003).
[14] C. Klempt *et al.*, Europhys. Journal D. **48**, 121 (2008).

Acknowledgments:
This work is funded by EPSRC grants F002068/1, G026602 SLC – Royal Society URF.